2023 - 2024

**Q2:** a) Adjacency matrix:

| | Hi | Je | Ma | Ja | E | Ch | A | S |
|---|---|---|---|---|---|---|---|---|
| Hi | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| Je | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Ma | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| Ja | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| E | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Ch | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| A | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| S | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |

b) $dc(\text{Jacob}) = \frac{3}{8-1} = \frac{3}{7} = 0.43$

$dc(\text{Sarah}) = \frac{1}{7} = 0.14$

$dc(\text{Emily}) = \frac{2}{7} = 0.29$

→ Jacob has the highest degree centrality.

c) $cc(\text{Matthew}) = \frac{7}{1+1+2+2+3+3+3} = \frac{7}{15} = 0.467$

$cc(\text{Ashley}) = \frac{7}{1+1+1+2+3+2+2} = \frac{7}{12} = 0.583$

$cc(\text{Micheal}) = \frac{7}{1+1+2+1+1+2+3} = \frac{7}{11} = 0.636$

→ Micheal has the highest closeness centrality.

Q3:

1) $cc(G) = \frac{1}{n} \sum_{i=1}^{n} cc(i)$ with $i$: node $\in G$

. $cc(\text{Micheal}) = \frac{2 \times N}{dc(dc-1)} = \frac{2 \times 1}{4(4-1)} = \frac{1}{6}$

. $cc(\text{Jessica}) = \frac{2 \times 0}{2(2-1)} = 0$

. $cc(\text{Matthew}) = 0$

. $cc(\text{Jacob}) = 0$

. $cc(\text{Emily}) = \frac{2 \times 1}{2(2-1)} = 1$

. $cc(\text{Christopher}) = \frac{2 \times 1}{3(3-1)} = \frac{1}{3}$

. $cc(\text{Ashley}) = 0$

. $cc(\text{Sarah}) = 0$

$\rightarrow cc(G) = \frac{1}{8} \times \left(\frac{1}{6} + 1 + \frac{1}{3}\right) = \frac{3}{16} = 0.1875$

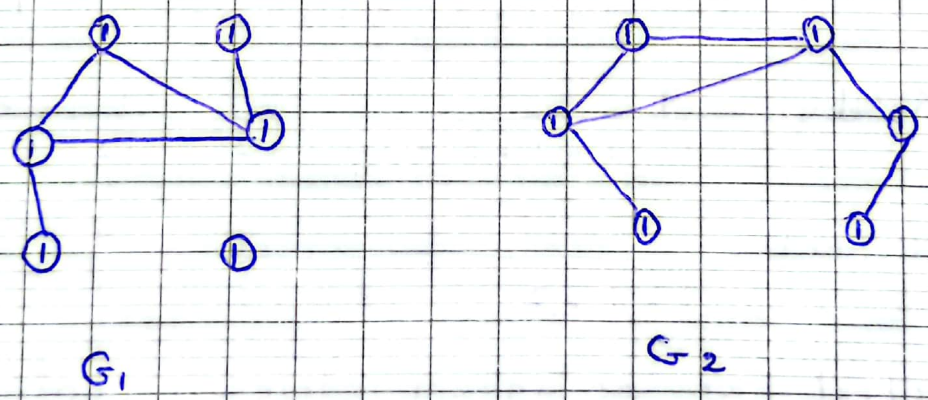
2). possible nb of edges: $K = \frac{n(n-1)}{2} = \frac{8(8-1)}{2} = 28$

. nb of current edges of $G$: $m = 10$

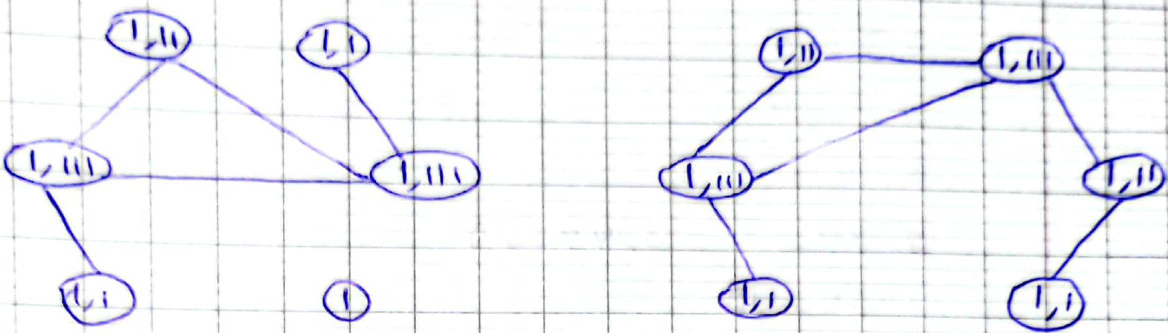
. possible nb of graphlets: $S = C(m, K) = C(10, 28)$

$\Rightarrow S = 13123110$

3). Assign initial colors:



• Aggregate neighboring colors:



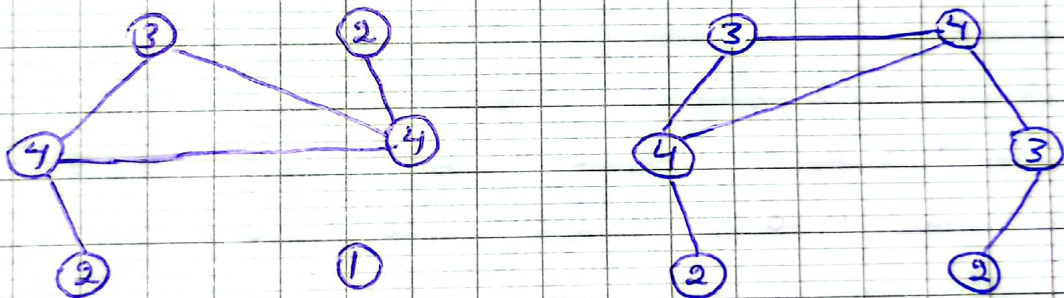

· hash aggregated colors:

1 → 1

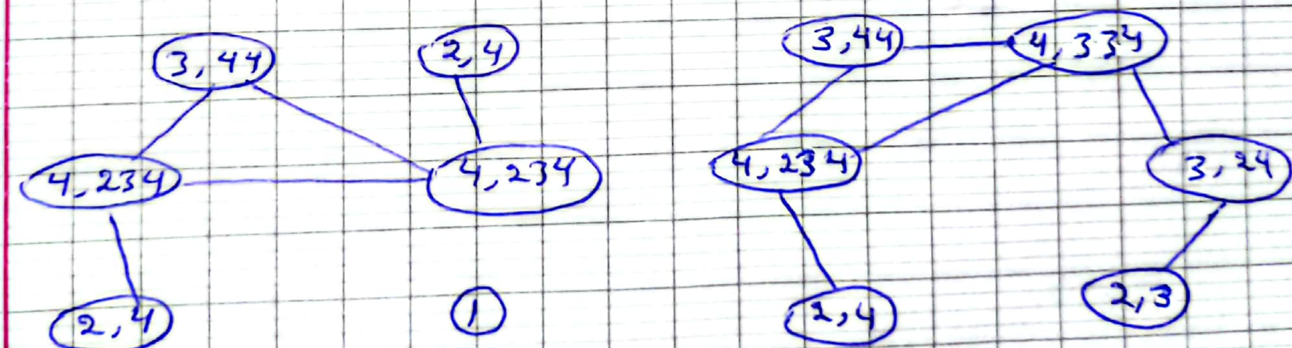
1,1 → 2

1,11 → 3

1,111 → 4




- Aggregate hashed colors:



- Hash aggregated colors:

2,3 → 5

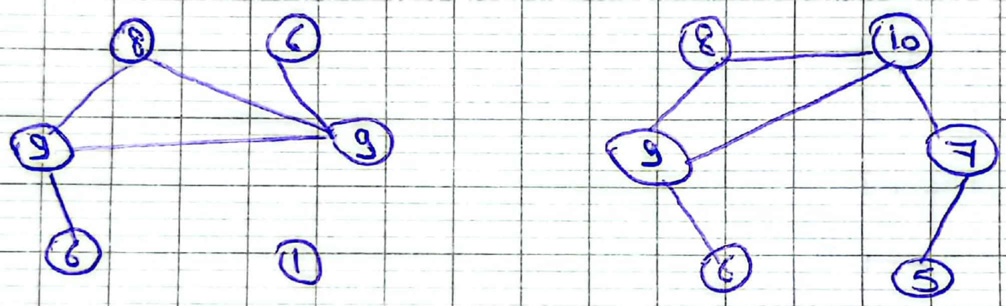
2,4 → 6

3,24 → 7

3,44 → 8

4,234 → 9

4,334 → 10



| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| colors: | 1, | 2, | 3, | 4, | 5, | 6, | 7, | 8, | 9, | 10, |
| $G^1$ count: | [6, | 2, | 1, | 2, | 0, | 2, | 0, | 1, | 2, | 0 ] |
| $G^2$ count: | [6, | 2, | 2, | 2, | 1, | 1, | 1, | 1, | 1, | 1 ] |

$$\rightarrow K(G^1, G^2) = \phi(G^1)^T \cdot \phi(G^2)$$

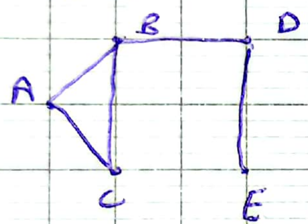
$$= 36 + 4 + 2 + 4 + 0 + 2 + 0 + 1 + 2 + 0$$

$$= 51$$

Q3: 4) Given 4 graphlets of size $k=3$

→ Graphlet kernel:
- Generate all possible $C(n,k)$ subsets of the graph
- For each subset, identify which graphlet type it matches.
- calculate the graphlet kernel
$$K(G^3, G^4) = \phi(G^3)^T \cdot \phi(G^4)$$

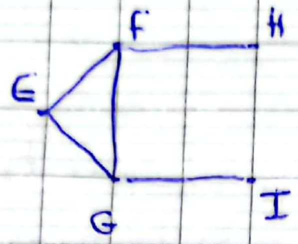
. For graph $G^3$:




possible subsets with their matching graphlets:
- (a, b, c): $g_1$
- (a, b, d): $g_2$
- (a, b, e): $g_3$
- (a, c, d): $g_3$
- (a, c, e): $g_3$
- (a, d, e): $g_3$
- (b, c, d): $g_2$
- (b, c, e): $g_3$
- (b, d, e): $g_2$
- (c, d, e): $g_3$

$$\Rightarrow \phi(G^3)^T = \begin{array}{cccc} g_1 & g_2 & g_3 & g_4 \\ \end{array}$$
$$\phi(G^3)^T = [\,1,\ 3,\ 6,\ 0\,]$$

- For $G^4$:




- $(e, f, g)$: $g_1$
- $(e, f, h)$: $g_2$
- $(e, f, i)$: $g_3$
- $(e, g, h)$: $g_3$
- $(e, g, i)$: $g_2$
- $(e, h, i)$: $g_4$
- $(f, g, h)$: $g_3$
- $(f, g, i)$: $g_2$
- $(f, h, i)$: $g_3$
- $(g, h, i)$: $g_3$

$$\Rightarrow \phi(G^4) = \begin{array}{cccc} g_1, & g_2, & g_3, & g_4 \\ \end{array}$$
$$\phi(G^4) = [1, \; 3, \; 5, \; 1]$$

- $K(G^3, G^4)$

$= \phi(G^3)^T \cdot \phi(G^4)$

$= 1 + 9 + 30 + 0$

$= 40$